Analysis for the Calendering of Bingham Plastic Fluids

INTRODUCTION

Calendering is a continuous process used to produce a sheet or film of uniform thickness. The theoretical analysis regarding the mechanism in calendering as it stands today was developed by Gaskell[1] and McKelvey[2] for Newtonian and power-law fluids. The analysis was extended by Brazinsky et al.[3] Alston and Astill[4] studied hyperbolic tangent viscosity model fluids. Kiparissides et al.,[5] using finite element methods, studied the flow behavior of Newtonian and non-Newtonian fluids. Dobbels et al.[6] and Kiparissides et al.[7] solved the nonisothermal case using orthogonal collocation and finite difference methods, respectively. Two comprehensive reviews concerning the isothermal case are also available in recent textbooks.[8,9]

The objective of this short note is to derive the flow mechanism of the calendering of Bingham plastic fluids and to investigate the effects of fluid physical properties on calendering operation.

MATHEMATICAL APPROACH

The governing equations for the conservation of mass and momentum in calendering have been developed elsewhere[2–5,8,9] using a lubrication approximation.[10] For a Bingham plastic material, the shear stress obeys

$$\tau_{yx} = \mu_0 \frac{du}{dy} \pm \tau_0 \qquad \text{if } |\tau_{yx}| > \tau_0 \tag{1}$$

$$\frac{du}{dy} = 0 \qquad \text{if } |\tau_{yx}| < \tau_0 \tag{2}$$

For symmetric calendering, the velocity profile can be obtained by solving the above equations with governing equations subjected to the boundary condition $U = U_0$ at $y = h(x)$:

$$U_x = U_0 + \frac{1}{2\mu}\frac{dP}{dx}[y^2 - h^2(x)] - \frac{\tau_0}{\mu}[y - h(x)] \qquad \text{for } y \geq y_0 \tag{3}$$

$$U_x = U_0 - \frac{1}{2\mu}\frac{dP}{dx}[y_0 - h(x)]^2 \qquad \text{for } y \leq y_0 \tag{4}$$

where P is the pressure in the gap region. With reference to Figure 1, U_0 is the velocity of roller, $h(x)$ is the distance from the central line to the roller surface, approximated[2,8] by

$$h(x) = H_0(1 + x^2/2H_0R) \tag{5}$$

y_0 is the upper boundary of the plug flow region, defined as

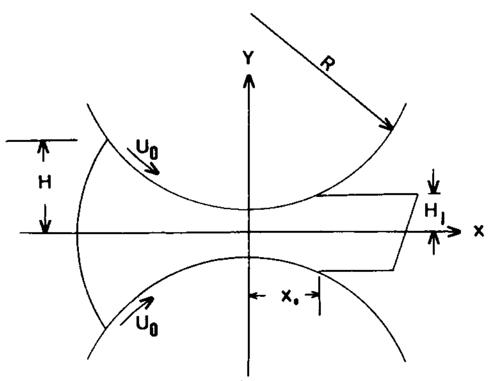

Fig. 1. Notations for the flow analysis in calendering.

Journal of Applied Polymer Science, Vol. 25, 967–970 (1980)

0021-8995/80/0025-0967$01.00

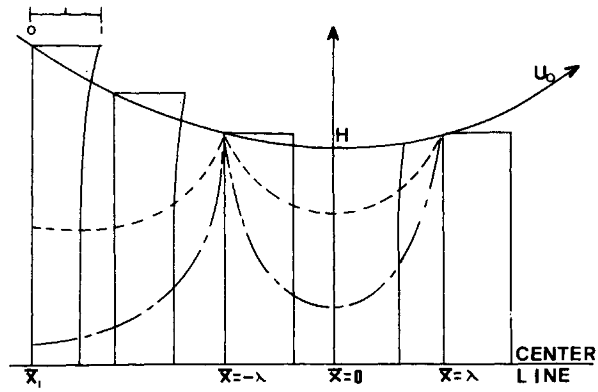

Fig. 2. Typical dimensionless velocity profiles. [$\lambda = 0.25$, $\tau_0 = 1 \times 10^8$ dyn/cm^2 (- - -), $\tau_0 = 1 \times 10^7$ dyn/cm^2 (——), $R = 10$ cm, $U = 20$ cm/sec, $H_0 = 0.001$ cm, $\mu = 1 \times 10^4$ P.]

$$y_0 \left|\frac{dP}{dx}\right| = \tau_0 \tag{6}$$

which is derived by momentum balance equation. In reality, the maximum value of y_0 should be $h(x)$.

If the sheet comes off the rollers with the same speed U_0, application of the mass continuity equation yields the following equation:

$$\frac{1}{\mu}\left(\frac{4}{3}y_0^3 - 2y_0^2 h(x) + \frac{2}{3}h(x)^3\right)\frac{dP}{dx} = 2U_0H_0\left(\frac{x^2}{2H_0R} - \lambda^2\right) + \frac{\tau_0}{2\mu}(h(x) - y_0)^2 \tag{7}$$

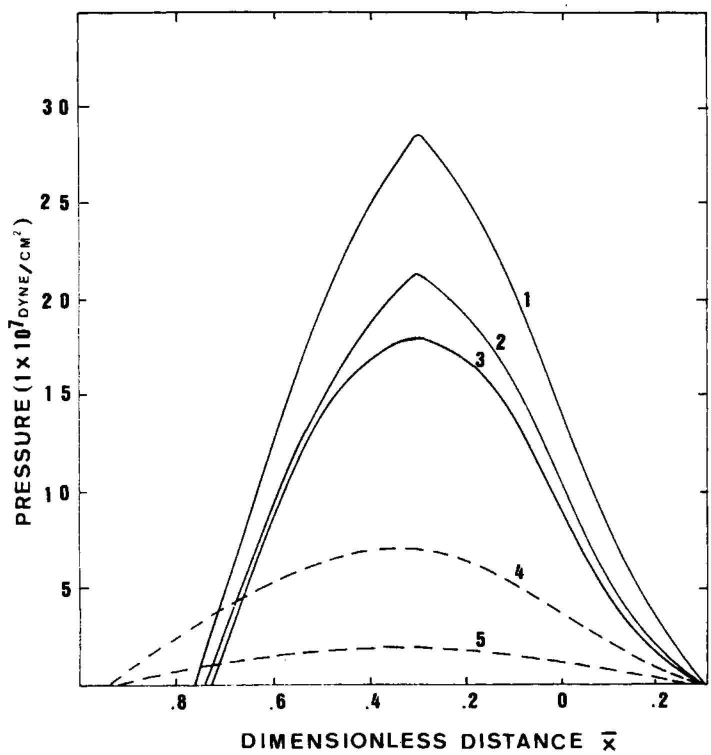

Fig. 3. Effects of τ_0 and power-law index n of the pressure profile. (1) τ_0: 5.0×10^6; (2) τ_0: 2.5×10^6; (3) τ_0: 0; (4) N: 0.75; (5) N: 0.5. ($\lambda = 0.3$, $R = 10$ cm, $U_0 = 40$ cm/sec, $H_0 = 0.01$ cm, $\mu = 1 \times 10^4$ P.)

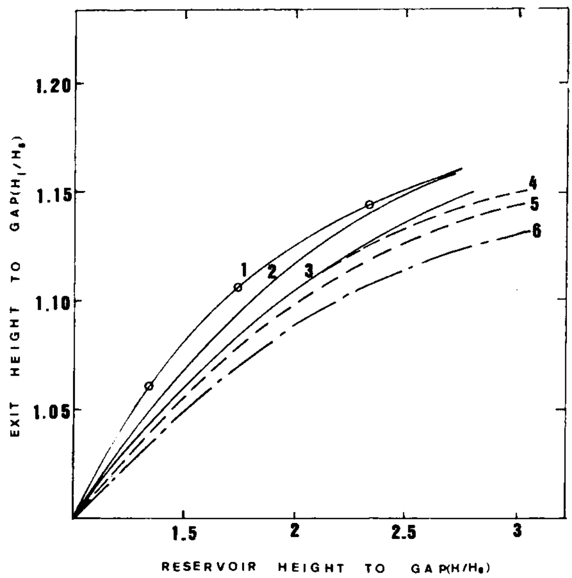

Fig. 4. Exit height (H_1/H_0) as a function of upstream reservoir thickness (H/H_0) for Newtonian (-⊖-) and non-Newtonian fluids. [Bingham plastic fluids (—): R = 10 cm, H_0 = 0.001 cm, $\tau_0/\mu U_0$ (dyn sec/P cm^3) = 250 (line 2) and 500 (line 3). Hyperbolic tangent model (- - -) of Alston and Astill: R = 1.25 cm, H_0 = 0.004 cm, U_0 (cm/sec) = 12.7 (line 4) and 50.8 (line 5). Power-law fluid (——): R = 10 cm, H_0 = 0.01 cm, n = 0.25.]

where

$$\lambda^2 = H/H_0 - 1 \tag{8}$$

Since y_0 is a function of dP/dx, eq. (7) is a nonlinear differential equation which can be solved using an iterative procedure. The pressure can be obtained by the integration of eq. (7) with the following boundary condition:

$$\text{at } x = x_0\text{:} \quad P = 0, \qquad U = U_0 \tag{9}$$

This boundary condition implies that $y_0 = h(x_0)$ at $x = x_0$. Therefore, the separation point of the sheet from the roller occurs at

$$x_0 = \lambda(2H_0R)^{1/2} \tag{10}$$

The result also applies to Newtonian and power-law fluids.[2,8,9]

RESULTS AND DISCUSSION

Some typical velocity profiles are shown in Figure 2. The dimensionless coordinate $\bar{x}$ is defined as

$$\bar{x} = x/(2H_0R)^{1/2}$$

Here $\bar{x}_1$ is the "contact" point where $P = 0$, and $\bar{x} = \lambda$ is the "leave-off" point where $P = 0$.[9] The velocity profiles around $\bar{x} = -\lambda$ are flat. The dotted lines shown in Figure 2 are the trace of y_0 for different τ_0. As τ_0 is increased, the dotted line position increases.

Figure 3 shows the effects of τ_0 and the power-law index on the pressure distribution in calendering. As τ_0 increases, the calender pressure increases. The total force required to clamp the rollers is the integral of the calender pressure over the entire gap region. Therefore, greater forces are required to clamp the rollers in calendering of the Bingham palstic fluids than the forces needed for Newtonian and power-law ($n \leq 1$) fluid systems. It is noteworthy that pressure derivative near $\bar{x} = -\lambda$ is so low that $y_0[= h(x)]$ and dP/dx cannot be calculated using eq. (8). However, this region is so narrow that the pressure curve is approximately symmetrical to the region where $\bar{x} = -\lambda$. Thus, we may neglect the integral that is in this region.

In Figure 4, H_1/H_0 (exit height to gap height) is plotted against H/H_0 (reservoir height to gap height) for Newtonian and non-Newtonian fluids. The calendering of Newtonian fluid is a special case of the calendering of power-law fluid ($n = 1$) and Bingham plastic fluid ($\tau_0 = 0$). When the ratio H/H_0 is kept constant, the calendering of Newtonian fluid produces higher values of H_1/H_0 than those produced by calendering of power-law fluid ($n \leq 1$) and Bingham plastic fluid. In addition, the relationship of H_1/H_0 vs. H/H_0 of a Newtonian fluid is independent of the roller speed, whereas it is dependent on the power-law index n for power-law fluids[5] and the parameter $\tau_0/\mu U_0$ for Bingham plastic fluids.

For hyperbolic tangent viscosity model fluids,[4] the slope of this curve (H_1/H_0 vs. H/H_0) decreases when the roller speed is increased, i.e., the thickness of the film produced becomes smaller at a higher roller speed. An interesting feature in calendering of the Bingham plastic fluids is that the slope of this curve is a monotonically decreasing function of the parameter $\tau_0/\mu U_0$. This fact implies that the sheet thickness becomes larger when the roller speed is increased. Furthermore, the viscosity of the Bingham plastic material has the same effect as velocity on calendering operation. The above analyses indicate that the material properties would influence quite strongly the operation variables in calendering processing.

The author thanks Professor M. E. Ryan for helpful discussions.

References

1. R. E. Gaskell, *J. Appl. Mech.*, **17**, 334 (1950).
2. J. M. McKelvey, *Polymer Processing*, Wiley, New York, 1962, Chap. 9.
3. I. Brazinsky, H. F. Cosway, C. F. Valle, Jr., R. Jones, R. Clark, and V. Story, *J. Appl. Polym. Sci.*, **14**, 2771 (1970).
4. W. W. Alston and K. N. Astill, *J. Appl. Polym. Sci.*, **17**, 3157 (1973).
5. C. Kiparissides and J. Vlachopoulos, *Polym. Eng. Sci.*, **16**, 719 (1976).
6. F. Dobbels and J. Mewis, *AIChE J.*, **23**, 224 (1977).
7. C. Kiparissides and J. Vlachopoulos, *Polym. Eng. Sci.*, **18**, 210 (1978).
8. S. Middleman, *Fundamentals of Polymer Processing*, McGraw-Hill, New York, 1977, p. 170.
9. Z. Tadmor and C. G. Gogos, *Principle of Polymer Processing*, Wiley, New York, 1979.
10. J. R. A. Pearson, *Mechanical Principles of Polymer Melt Processing*, Pergamon, New York, 1966, p. 60.

T.-S. CHUNG

Department of Chemical Engineering
State University of New York at Buffalo
Amherst, New York 14260

Received August 27, 1979
Revised December 13, 1979